Calculus Warm Up #4-5

For: $s(t) = 2t^2 - 5$

1. Find the average rate of change on the interval [-3, 2]

2. Find the instantaneous rate of change at each endpoint of the interval.

AP Calculus AB
AP Problem Set #1

(No Calculator)

1 | 1 0 −7 6
1 1 −6
1 1 −6 0

1. Let f be the function given by $f(x) = x^3 - 7x + 6$.

$0 = x^3 - 7x + 6$

(a) Find the zeros of f.

(b) Write an equation of the line tangent to the graph of f at $x = -1$.

$-3, 1, 2$

$f'(-1) =$

$y - 12 = -4(x + 1)$

2. Consider the curve defined by $x^2 + xy + y^2 = 27$.

(a) Write an expression for the slope of the curve at any point (x, y).

(b) Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

(c) Find the points on the curve where the lines tangent to the curve are vertical.

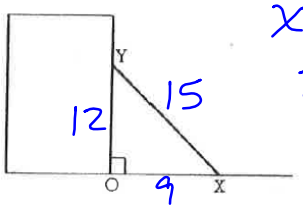
a) $\frac{dy}{dx} = -\frac{2x+y}{x+2y}$

b) $x = \pm 3\sqrt{3}$; both slopes $= -2$
(x-intercepts) (put into $\frac{dy}{dx}$)

3. A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

(a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.

(b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.



$x^2 + y^2 = 15^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(9)\frac{1}{2} + 2(12)\frac{dy}{dt} = 0$

$\frac{dx}{dt} = \frac{1}{2}$ when $x = 9$, $y = 12$

a) $\frac{dy}{dt} = -\frac{3}{8}$ ft/sec

b) $\frac{dA}{dt} = \frac{21}{16}$ ft^2/sec

$A = \left(\frac{1}{2}x\right)y$

$\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + \frac{1}{2}\frac{dx}{dt}\cdot y$

Chapter 4 Applications of Differentiation

4.1 Extrema:

Relative extrema and absolute extrema

Critical Numbers

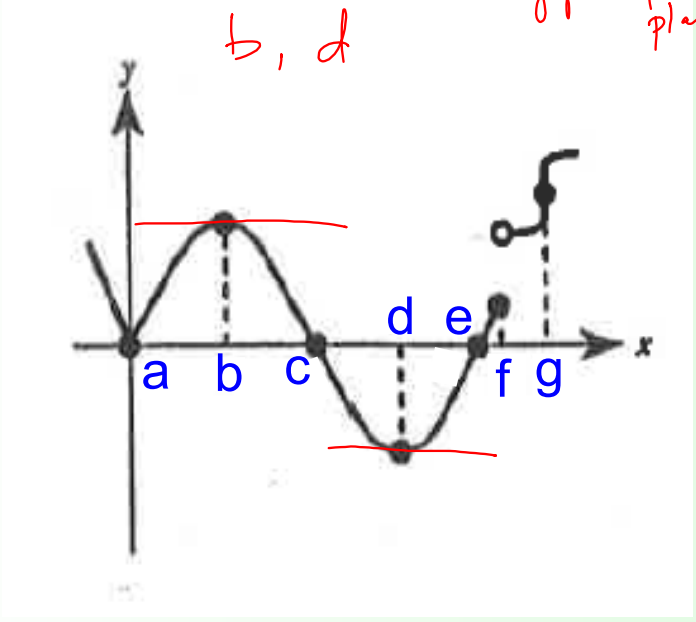
Where is the function not differentiable?

g vertical tangent, f, a

Where is the derivative zero?

gap, pointy place

b, d

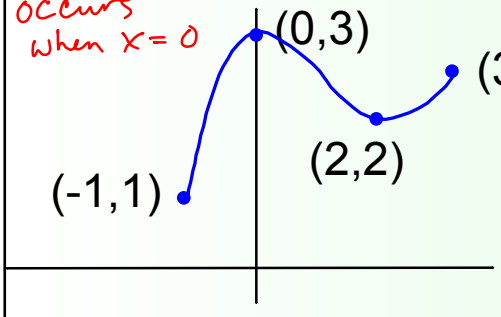


Extrema:

The exteme values of a function.
(outcomes, y-values)

Finding extrema on a closed interval: Looking for the highest and lowest points on that interval. The extrema will be the y-values there.

highest point: (0,3)
extrema is 3
occurs when x = 0



Closed interval: [-1, 3]
x values

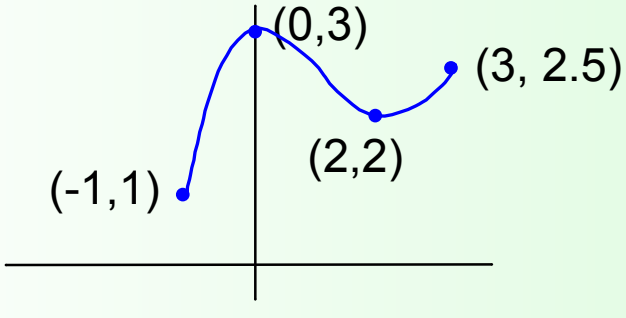
lowest y
point (-1,1)
extrema is 1 occurs at x = -1

Absolute Extrema

The highest or the lowest point on the interval including the endpoints

Relative Extrema

Other places on the interval where the derivative is zero or undefined.



Find the extrema of $f(x) = x^2 + 2$ on $[-1, 4]$

1) See where $f'(x) = 0$ or is undefined.

$f'(x) = 2x$

$0 = 2x$

$x = 0$

$f(0) = 0^2 + 2$

$f(0) = 2 \rightarrow (0, 2)$ ← y

2) Check endpoints.

$f(-1) = (-1)^2 + 2 = 3 \rightarrow (-1, 3)$ ← y

$f(4) = 4^2 + 2 = 18 \rightarrow (4, 18)$ ← y

3) Compare all the outcomes, y, for highest and lowest value.

Absolute Extrema absol.

$= 18$ max

$= 2$ absolute min

Theorems and Definitions to understand:

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THEOREM 4.1 THE EXTREME VALUE THEOREM

If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

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DEFINITION OF CRITICAL NUMBER

If f is defined at c, then c is called a **critical number** of f if $f'(c) = 0$ or if f' is undefined at c.

→ x-values where the derivative is zero or undefined.

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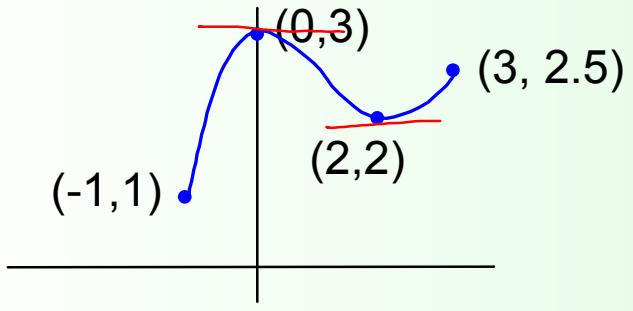
THEOREM 4.2 RELATIVE EXTREMA OCCUR ONLY AT CRITICAL NUMBERS

If f has a relative minimum or relative maximum at $x = c$, then c is a critical number of f.

Critical Numbers of f,
(the x values where f′(x) = 0 or is undefined)

$x = 0, 2 \quad f'(x) = 0$

f(x):



Find the extrema of $f(x) = 3x^4 - 4x^3$ on [-1, 2]

1) Find the critical numbers.
(Find x where f′(x) = 0 or is undefined.)

$f'(x) = 12x^3 - 12x^2$

$0 = 12x^3 - 12x^2$

$= 12x^2(x-1)$ → $x = 0, 1$

2) Evaluate f at critical numbers and endpoints.
(Find corresponding y-values.)

$f(0) = 0$ $\quad f(-1) = 7$ $\quad$ Min. @ $x = 1$

$f(1) = -1$ $\quad f(2) = 16$ $\quad$ Extrema (Minimum Value) $= -1$

3) Compare all the outcomes, y, for highest and lowest value.

Maximum @ $x = 2$
Extrema = 16
(Max value)

Which are absolute and which are relative?

Find the extrema of $f(x) = 2x - 3x^{2/3}$ on the interval $[-1, 3]$.

$f'(x) = 2 - \frac{2}{\sqrt[3]{x}}$

undefined @ $x = 0$

$0 = 2 - \frac{2}{\sqrt[3]{x}}$

$x = 1$

critical #'s

$x = 0, 1$

Evaluate @ critical #'s & endpts.

$f(0) = 0$

$f(1) = -1$

$f(-1) = -5$

$f(3) \approx -0.24$

HW: p. 160 # 1 - 25 odd